Boone's Finance Boot Camp

Series 1 Companion Notes: Time Value of Money

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Contents

1 How To Use These Notes

These notes go with Series 1 of Boone's Finance Boot Camp. The videos are short on purpose. They give you the move, show the move, and then ask you to practice. These notes give you a little more room to slow down.

Think of the videos as the fast first pass and these notes as the fuller second pass. In the videos, we keep momentum. Here, we can define terms more carefully, work more examples, and linger over the places where students commonly get tripped up.

Use them this way:

1. Watch the matching video.
2. Read the matching section here.
3. Work the quick checks before looking at the answers.
4. Use the quizzes for randomized practice and feedback.
5. Use the Excel workbooks when a spreadsheet is the right tool.

The goal is not to memorize a pile of formulas. The goal is to build a few habits: draw the timeline, match the rate to the period, keep signs straight, and discount cash flows to the right date.

For self-study, try the questions before looking at the answers. The answers are collected in an answer key near the back of these notes so that the solution is not sitting directly under the question.

The Core Habit

Before using a formula, ask three questions:
When does each cash flow happen?
What sign should each cash flow have from my perspective?
Does the rate match the period on the timeline?

1.1 Series Map

Videos	Main job	Practice checkpoint
S1-01 to S1-03	Timelines, PV/FV, APR/EAR	Quiz S1-A
S1-04 to S1-05	Solve for `r` or `N`, annuities, perpetuities	Quiz S1-B
S1-06 to S1-09	Bonds, calculator pricing, Excel pricing, YTM	Quiz S1-C
S1-10 to S1-11	Irregular NPV and terminal value timing	Quiz S1-D

2 Timelines And Sign Convention

Learning goal. Draw a simple timeline, place cash flows at the right dates, and use signs from one clear perspective.

Relevant video. S1-01.

The timeline is the backbone of time value of money. If the timeline is wrong, the formula can be perfect and the answer can still be wrong. That is not a fun place to be.

A timeline does not need to be fancy. It just needs to answer the basic timing question: what happens now, what happens later, and how far apart are those cash flows? In finance, a surprising number of mistakes come from skipping that simple picture.

2.1 Plain-English Principle

Pick one perspective. Cash you receive is positive. Cash you pay is negative. Then place each cash flow at the date it happens.

Sign Convention

From one clear perspective:

$$\text{cash in} = +$$

$$\text{cash out} = -$$

The signs are not moral judgments. Negative does not mean bad. It just means cash is leaving from the perspective you chose.

2.2 Worked Example

You invest $100 today and receive $110 one year from now.

From your perspective:

$$t = 0 : -100$$

$$t = 1 : +110$$

The investment is cash out today. The payoff is cash in later.

2.2.1 Realistic Example

A student puts $500 into a certificate of deposit today and expects $540 one year from now. The timeline is not complicated:

$$t = 0 : -500, \qquad t = 1 : +540$$

That little sign habit is the same habit we will use later for bonds, projects, and valuation.

2.2.2 Common Mistake

Do not switch perspectives halfway through the problem. If the borrower receives money today and repays later, the borrower sees cash in today and cash out later. The lender sees the opposite.

2.2.3 Go a little further

The same cash flow can be positive or negative depending on perspective. That is why financial calculators often return a negative present value when future payments are positive. The calculator is not upset with you. It is enforcing a cash-flow sign convention.

2.3 Quick Checks

1. You pay \$250 today and receive \$300 in two years. What are the signs?
2. A bank lends you \$1,000 today and you repay \$1,100 in one year. From your perspective, what are the signs?

Answers are in the answer key at the back.

2.3.1 Tiny self-test

If you can explain why the lender and borrower use opposite signs for the same loan, you understand sign convention well enough to move forward. If that feels slippery, draw the cash flows twice: once from the lender's perspective and once from the borrower's perspective.

3 Present Value, Future Value, And Rates

Learning goal. Move one cash flow forward or backward in time, and make sure the rate matches the timeline.

Relevant videos. S1-02 and S1-03.

This is the financial time travel section. Once you can move one cash flow through time, a lot of finance becomes less mysterious.

The language can sound bigger than the idea. Future value asks, "What will this become later?" Present value asks, "What is that future amount worth today?" Once you are comfortable moving one cash flow, you are ready to move many cash flows.

3.1 Plain-English Principle

Future value moves money forward. Present value moves money backward.

PV And FV Formulas

For one cash flow:

$$FV = PV(1+r)^N$$

$$PV = \frac{FV}{(1+r)^N}$$

Use r as the rate per period and N as the number of periods.

3.2 Worked Example

If you invest $1,000 for 3 years at 8% per year:

$$FV = 1{,}000(1.08)^3 = 1{,}259.71$$

If you need $1,000 in 3 years and the discount rate is 8%:

$$PV = \frac{1{,}000}{(1.08)^3} = 793.83$$

Same relationship. Different direction.

APR And EAR

An APR is a quoted annual rate. It may need to be converted before you use it.
If compounding happens m times per year:

$$EAR = \left(1 + \frac{APR}{m}\right)^m - 1$$

The period rate is:

$$\frac{APR}{m}$$

3.3 Compounding Example

A 12% APR compounded monthly does not earn exactly 12% effectively over the year.

Monthly rate:

$$\frac{0.12}{12} = 0.01$$

Effective annual rate:

$$(1.01)^{12} - 1 = 12.68\%$$

In plain English: monthly compounding lets interest earn interest during the year.

3.4 Rate And Timeline Match

Timeline	Rate that belongs there	Example
Annual	annual rate	one cash flow per year
Semiannual	semiannual rate	most coupon bonds

Timeline	Rate that belongs there	Example
Monthly	monthly rate	monthly loan payments
Daily	daily rate	some bank or brokerage calculations

When in doubt, write the period labels first. If the timeline says months, the rate should be monthly. If the timeline says six-month periods, the rate should be semiannual.

3.4.1 Common Mistake

Do not use an annual rate with monthly periods unless the problem explicitly says that is what you should do. If cash flows are monthly, use a monthly rate. If cash flows are semiannual, use a semiannual rate.

3.4.2 Realistic Example

A store offers "12 months same as cash" financing, but the fine print quotes a monthly rate after the promotional period. The timing matters. A monthly rate belongs on a monthly timeline, not an annual one.

3.4.3 Go a little further

APR and EAR are two ways of describing rates. The APR is convenient for quoting. The EAR is convenient for comparing. If two loans have different compounding frequencies, compare their EARs before deciding which is more expensive.

3.5 Quick Checks

1. What is the future value of $500 in 4 years at 6% per year?
2. What is the present value of $750 received in 2 years at 10% per year?
3. A 6% APR compounded monthly has what monthly rate?

Answers are in the answer key at the back.

3.5.1 Why compounding frequency matters

Compounding frequency matters most when the quote and the cash-flow timing do not match. A rate can look small per month and still become meaningfully larger over a year because interest earns interest. That is the entire EAR idea in one sentence.

4 Solving For Unknowns And Repeated Cash Flows

Learning goal. Solve for the missing rate or number of periods, then use shortcuts for repeated cash flows.

Relevant videos. S1-04 and S1-05.

The same TVM structure can be turned around. Sometimes you know today and tomorrow, and you need the rate. Sometimes you know the rate and need the time.

This is also where the course starts to reward pattern recognition. If there is only one cash flow, use the single-cash-flow formulas. If the same cash flow repeats, look for an annuity or perpetuity shortcut. If the cash flows are uneven, do not force a shortcut.

4.1 Solving For r Or N

Solving The Single-Cash-Flow Formula

If you know PV, FV, and N, solve for r:

$$r = \left(\frac{FV}{PV}\right)^{1/N} - 1$$

If you know PV, FV, and r, solve for N:

$$N = \frac{\ln(FV/PV)}{\ln(1+r)}$$

4.2 Worked Example

$100 grows to $150 in 5 years.

$$r = \left(\frac{150}{100}\right)^{1/5} - 1 = 8.45\%$$

At about 8.45% per year, $100 becomes $150 over 5 years.

4.2.1 Common Mistake

Do not forget that N is the number of periods, not always the number of years. Semiannual bond periods and monthly loan periods are not annual periods.

4.3 Annuities And Perpetuities

An annuity is a level cash flow for a fixed number of periods. A perpetuity is a level cash flow forever. A growing perpetuity is a cash flow that grows at a constant rate forever.

Repeated Cash-Flow Shortcuts

Ordinary annuity present value:

$$PV = C\left[\frac{1 - \frac{1}{(1+r)^N}}{r}\right]$$

Level perpetuity present value:

$$PV = \frac{C_1}{r}$$

Growing perpetuity present value:

$$PV = \frac{C_1}{r - g}$$

In the perpetuity formulas, C_1 is the next period’s cash flow.

4.4 Worked Example

You will receive $100 per year for 5 years. The discount rate is 8%.

$$PV = 100\left[\frac{1 - \frac{1}{(1.08)^5}}{0.08}\right] = 399.27$$

A growing perpetuity will pay $100 next year, grow at 3% per year, and the discount rate is 8%.

$$PV = \frac{100}{0.08 - 0.03} = 2{,}000$$

4.5 Shortcut Selection

Cash-flow pattern	Useful tool
One cash flow	PV or FV formula
Same cash flow for fixed periods	Ordinary annuity formula
Same cash flow forever	Level perpetuity formula
Growing cash flow forever	Growing perpetuity formula
Uneven cash flows	Discount each cash flow separately

This is a recognition game. The faster you recognize the pattern, the faster you know which tool belongs.

4.5.1 Realistic Example

A scholarship fund wants to pay a fixed scholarship every year. If the payment is intended to continue forever, a perpetuity shortcut gives a first estimate of how much money the fund needs today.

4.5.2 Go a little further

The annuity formula is just a shortcut for discounting each cash flow separately and adding the present values. If you ever distrust the shortcut, build the timeline and discount each payment one at a time. You should land in the same neighborhood, with small differences only from rounding.

4.6 Quick Checks

1. What is the PV of $200 per year for 3 years at 10%?
2. What is the PV of a level perpetuity paying $50 next year if $r = 5\%$?
3. In the growing perpetuity formula, does C_1 happen today or next period?

Answers are in the answer key at the back.

4.6.1 Excel note

Excel's `PV` function can check an annuity, but it also uses sign convention. If the payment is entered as positive, Excel may return a negative present value. That does not mean the math failed. It means Excel is treating one side of the transaction as cash in and the other as cash out.

5 Bonds

Learning goal. Treat a bond as coupons plus face value, then price it using semiannual inputs.

Relevant videos. S1-06, S1-07, S1-08, and S1-09.

A plain-vanilla bond is not magic. It is a stream of coupon payments plus the face value at maturity.

The bond videos split the topic into pieces. First, see what a bond is: coupons plus face value. Then practice pricing with a financial calculator. Then practice pricing in Excel. Finally, turn the problem around and solve for YTM. Those are separate skills, but they are all built on the same timeline.

5.1 Plain-English Principle

The bond price is the present value of the promised cash flows.

Semiannual Bond Inputs

For a standard coupon bond with semiannual coupons:

$$N = \text{years to maturity} \times 2$$

$$i = \frac{YTM}{2}$$

$$PMT = \frac{\text{annual coupon rate} \times \text{face value}}{2}$$

5.2 Worked Example

A bond has:

- face value = $1,000,
- annual coupon rate = 6%,
- maturity = 5 years,

- YTM = 8%,
- semiannual coupons.

Then:

$$N = 10$$

$$i = 4\%$$

$$PMT = 30$$

The price is:

$$PV = 30\left[\frac{1 - \frac{1}{(1.04)^{10}}}{0.04}\right] + \frac{1{,}000}{(1.04)^{10}} = 918.89$$

The bond sells below par because the coupon rate is lower than the YTM.

5.2.1 Common Mistake

Do not put the annual coupon payment into a semiannual bond problem. A 6% coupon on $1,000 is $60 per year, but it is $30 every six months.

5.2.2 Realistic Example

A city issues bonds to pay for a new facility. Investors do not just look at the coupon. They compare the bond's promised cash flows to the return required for bonds with similar risk and maturity.

5.3 Calculator And Excel Habits

On a financial calculator, the example inputs are:

$$N = 10, \quad i = 4, \quad PMT = 30, \quad FV = 1{,}000$$

Solve for PV. A negative display is usually sign convention.

In Excel, build the cash-flow timeline, discount each cash flow, and sum the present values. Excel's $PV()$ function is a useful check, but the timeline is the habit.

5.4 Bond Price Intuition

Relationship	Price intuition
Coupon rate = YTM	price near par
Coupon rate < YTM	price below par
Coupon rate > YTM	price above par

The coupon rate sets the promised coupon. The YTM is the market-required return. Price moves so those two can live together.

5.4.1 Go a little further

YTM is the single discount rate that makes the present value of the bond's promised cash flows equal its observed price. That is why Excel Goal Seek is helpful: set calculated price equal to observed price by changing the periodic yield.

5.5 Quick Checks

1. An 8% coupon bond with $1,000 face value pays semiannually. What is the semiannual coupon?
2. A 3-year semiannual bond has how many periods?
3. If coupon rate is below YTM, should price usually be above or below par?

Answers are in the answer key at the back.

5.5.1 YTM is an internal rate

YTM is not a promised reinvestment plan. It is the discount rate that makes the present value of the bond cash flows equal the observed price. In that sense, it works like an internal rate of return for the bond's promised cash-flow stream.

6 Irregular NPV And Terminal Value

Learning goal. Lay out irregular cash flows, discount each one, and handle terminal value timing carefully.

Relevant videos. S1-10 and S1-11.

This is where the earlier habits start to feel big. Projects, investments, and valuations often have cash flows that do not follow a neat annuity pattern.

There is not much new math here. That is the nice part. We are still moving cash flows through time. The difference is that the cash flows are uneven, so we stop looking for a shortcut and build the timeline directly.

6.1 Net Present Value

NPV is the value today of all project cash flows, including the time-zero investment.

Manual NPV

For cash flows at different dates:

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_N}{(1+r)^N}$$

Do not discount CF_0. It is already today.

6.2 Worked Example

A project costs $500 today and pays $200, $250, and $300 over the next three years. The discount rate is 10%.

$$NPV = -500 + \frac{200}{1.10} + \frac{250}{1.10^2} + \frac{300}{1.10^3}$$

$$NPV = 113.82$$

Positive NPV means the project is expected to create value at that required return.

6.3 Manual Excel Workflow

For irregular cash flows, make the spreadsheet look like the timeline:

Row	Job
Time	label `0, 1, 2, 3...`
Cash flow	enter the cash flow at each date
Discount factor	compute `1/(1+r)^t`
Present value	cash flow times discount factor
NPV	sum the present values

When copying formulas across columns, use `F4` to lock the rate cell.

6.3.1 Common Mistake

Excel's $NPV()$ function discounts the first value in the range as if it happens one period from now. If you use it, do $NPV(rate, future\ cash\ flows) + CF_0$. In these videos, the main habit is manual discounting.

6.4 Terminal Value

Sometimes a project has explicit cash flows for a few years and then a continuing cash flow after that. The growing perpetuity shortcut helps.

Terminal Value Timing

If the first continuing cash flow is C_4, then the terminal value is placed at $t = 3$:

$$TV_3 = \frac{C_4}{r - g}$$

Then discount TV_3 back to today with the other $t = 3$ cash flow.

6.5 Worked Example

Suppose a project has:

$$CF_0 = -1{,}000, \quad CF_1 = 250, \quad CF_2 = 300, \quad CF_3 = 350$$

After year 3, the first continuing cash flow is $250 at $t = 4$, growing at 2% forever. The discount rate is 10%.

Terminal value at $t = 3$:

$$TV_3 = \frac{250}{0.10 - 0.02} = 3{,}125$$

Then the $t = 3$ amount is:

$$350 + 3{,}125 = 3{,}475$$

Now discount the $t = 1$, $t = 2$, and $t = 3$ amounts back to today.

6.5.1 Realistic Example

A small business might forecast detailed cash flows for three years because those numbers are concrete. After that, the forecast becomes less precise, so analysts often use a continuing-value estimate rather than pretending to forecast every year forever.

6.5.2 Go a little further

You could draw the growing cash flow out for many years and discount every amount. If the growth rate is less than the discount rate, the present value converges toward the growing perpetuity shortcut. The shortcut is not magic. It is a compact version of a very long discount-and-sum table.

6.6 Quick Checks

1. A project has $CF_0 = -100$, $CF_1 = 60$, and $CF_2 = 60$. At 10%, what is NPV?
2. A continuing cash flow starts at $t = 5$. At what date is the terminal value placed?
3. If a terminal value is placed one year too late, what happens to the valuation?

Answers are in the answer key at the back.

6.6.1 Terminal value reality check

Terminal value can be a large part of a valuation. That does not automatically make it wrong, but it does mean timing, growth, and discount-rate assumptions deserve extra care. A tiny change in long-run growth can move the answer a lot.

7 How The TVM Pieces Fit Together

Relevant videos. S1-01 through S1-11.

This section is a slower conceptual bridge. If the formulas feel like separate tools, this is the place to pause and see the system.

Time value of money is built on one claim: a dollar at one date is not the same as a dollar at another date. A dollar today can be invested. A dollar in the future is uncertain, delayed, or at least not available for use right now. Finance gives us a disciplined way to compare those dollars.

The discipline is not complicated, but it is strict. We cannot add cash flows until they are measured at the same date. We cannot use a monthly rate on an annual timeline. We cannot use the annual coupon payment when the bond pays semiannual coupons. Those rules may feel picky, but they are what make the answer mean something.

7.1 One Big Idea: Put Everything On The Same Date

Most of Series 1 can be summarized like this:

1. Identify the cash flows.
2. Put each cash flow on the timeline.
3. Match the rate to the period.
4. Move the cash flows to the date where you want the answer.
5. Add the values only after they are at the same date.

That is it. Present value moves cash flows backward. Future value moves cash flows forward. NPV moves all project cash flows to today and then adds them. Bond pricing moves all coupon and face-value cash flows to today. Terminal value uses a shortcut for a long future stream and then moves that shortcut value to the correct date.

Different chapter titles, same basic move.

7.2 What The Discount Rate Is Doing

The discount rate is the required return for the cash flow. If the discount rate is 10%, we are saying that a future cash flow must be adjusted for the fact that we require 10% per period to wait for it.

For one future cash flow:

$$PV = \frac{FV}{(1+r)^N}$$

The denominator is doing the discounting. If r rises, the denominator gets larger and the present value falls. If N rises, the denominator also gets larger and the present value falls. That is why distant cash flows are worth less today, especially at higher discount rates.

This is useful intuition on exams. Before calculating, ask: should the answer be bigger or smaller? If the discount rate goes up, PV should go down. If the future cash flow is farther away, PV should go down. If your answer moves the wrong way, something in the setup is probably off.

7.3 Why Shortcuts Are Allowed

Annuities and perpetuities are shortcuts, but they are not shortcuts around the logic. They are shortcuts through the arithmetic.

An ordinary annuity formula is a compact way to discount each equal payment and add the present values. A perpetuity formula is a compact way to value a cash flow stream that continues forever. A growing perpetuity is the same idea with constant growth.

The shortcut is allowed because the pattern is regular. If the pattern is not regular, do not use the shortcut. Just discount each cash flow separately.

Pattern	Why shortcut works
Same cash flow for fixed periods	equal payments create an annuity pattern
Same cash flow forever	infinite level stream creates a perpetuity pattern
Cash flow grows at constant rate forever	growth is regular enough for the growing perpetuity formula
Uneven cash flows	no regular shortcut, so discount one at a time

This is why pattern recognition matters. A formula is not just something to memorize. A formula is a tool for a specific cash-flow pattern.

7.4 Bonds Are Not A New Universe

Bonds can feel like a new topic because of the vocabulary: coupon, face value, maturity, YTM, premium, discount. But the math is still present value.

A plain coupon bond has two pieces:

1. a coupon annuity,
2. a face value paid at maturity.

That means bond price is:

$$\text{PV of coupons} + \text{PV of face value}$$

The semiannual convention is the main mechanical issue. If a bond pays coupons semiannually, the timeline is in six-month periods. The periodic coupon, periodic yield, and number of periods all need to match that six-month timeline.

Once that is clear, the price intuition becomes easier:

- if the coupon rate is below YTM, the bond sells below par;
- if the coupon rate equals YTM, the bond sells near par;

- if the coupon rate is above YTM, the bond sells above par.

Price adjusts so investors can earn the required return on the promised cash flows.

7.5 NPV Is The Same Habit With A Decision Attached

NPV adds one more layer: interpretation. We are no longer just asking, "What is this cash flow worth?" We are asking, "Does this project create value?"

A positive NPV means the project is expected to create value after earning the required return. A negative NPV means the project is expected to fall short of the required return. A zero NPV means the project exactly earns the required return.

That is why CF_0 matters. If a project costs money today, that cost is part of the value calculation. The future cash flows may be positive, but the project may still fail to create value after considering timing, risk, and the upfront investment.

7.5.1 A Fuller Intuition Check

Suppose two projects both pay $1,000 total over the next few years. One pays most of it early. The other pays most of it later. The early project is usually more valuable because the cash arrives sooner.

Suppose two projects have identical cash-flow timing, but one is riskier. The riskier project should usually have a higher discount rate. That higher rate lowers present value.

Suppose a project has a huge terminal value. That may be reasonable, but it means the long-run assumptions deserve attention. Terminal value is a shortcut for many future cash flows, so small changes in long-run growth and discount rates can matter a lot.

The point is not to be suspicious of every answer. The point is to understand what the answer depends on.

8 Extended Practice Lab

This section gives you a little more room to practice. The videos move quickly on purpose. Here, we can slow down, show more steps, and connect the pieces.

Relevant videos. S1-01 through S1-11.

The main exam habit is still the same: timeline first, formula second.

8.1 Lab 1: Timeline And Sign Discipline

Suppose you borrow \$2,000 today and repay \$2,240 one year from now.

From your perspective as the borrower:

$$t = 0 : +2{,}000$$

$$t = 1 : -2{,}240$$

From the lender’s perspective:

$$t = 0 : -2{,}000$$

$$t = 1 : +2{,}240$$

Both timelines describe the same contract. They just use different perspectives. This is why signs can feel a little strange at first. The signs are not describing whether the loan is good or bad. They are describing cash direction.

8.1.1 Practice Move

Write the perspective before the numbers:

Situation	Perspective	Today	Later
You invest in a CD	investor	cash out	cash in
You take out a loan	borrower	cash in	cash out
You buy a bond	bond investor	cash out	coupons and face value in
A company starts a project	company	investment out	project FCF in

That one extra word, perspective, prevents a lot of mistakes.

8.1.2 Go a little further

Many finance calculator problems require one side of the transaction to be negative and the other to be positive. If all cash flows have the same sign, the calculator may refuse to solve or give a confusing result. That is not the calculator being dramatic. It is telling you the cash-flow pattern does not look like an exchange.

8.2 Lab 2: Financial Time Travel With One Cash Flow

Take $700 today and move it forward 6 years at 7%.

$$FV = 700(1.07)^6 = 1{,}050.51$$

Now take $1,050.51 six years from now and move it backward at the same rate.

$$PV = \frac{1{,}050.51}{(1.07)^6} = 700.00$$

That is the whole idea. The future value formula and present value formula are the same relationship, just used in opposite directions.

8.2.1 A Slightly More Realistic Twist

You want $5,000 available in 4 years for a study-abroad trip. Your savings account is expected to earn 5% per year.

How much do you need today?

$$PV = \frac{5{,}000}{(1.05)^4} = 4{,}113.51$$

So the answer is not "save $5,000 today." If the account really earns 5% per year, $4,113.51 today grows to about $5,000 in 4 years.

8.2.2 Common Mistake

Do not confuse the amount you want in the future with the amount you need today. Present value is usually smaller than future value when the discount rate is positive.

8.3 Lab 3: APR, EAR, And Matching The Period

Suppose a loan quotes 9% APR compounded monthly.

Monthly rate:

$$\frac{0.09}{12} = 0.0075 = 0.75\%$$

EAR:

$$(1.0075)^{12} - 1 = 9.38\%$$

If a problem has monthly cash flows, use 0.75% per month. If you are comparing this loan to another annual opportunity, the 9.38% EAR is the cleaner comparison.

8.3.1 Semiannual Rate Practice

Bonds usually pay coupons semiannually. If a bond has an 8% quoted annual YTM, the semiannual discount rate is:

$$\frac{8\%}{2} = 4\%$$

If the bond matures in 7 years, the number of semiannual periods is:

$$7 \times 2 = 14$$

So the bond problem uses:

$$N = 14, \qquad i = 4\%$$

8.3.2 Rate Matching Table

Quote or cash-flow pattern	Period rate	Number of periods
12% APR, monthly, 18 months	1%	18
8% annual rate, 5 years	8%	5
10% quoted YTM, semiannual, 6 years	5%	12
6% APR, quarterly, 2 years	1.5%	8

8.3.3 Go a little further

An EAR is especially useful when two quoted APRs have different compounding frequencies. A 10% APR compounded annually and a 10% APR compounded monthly are not exactly the same economic rate. Monthly compounding earns interest on interest inside the year.

8.4 Lab 4: Repeated Cash Flows Without Losing The Timeline

An annuity shortcut saves time, but it is still just a shortcut for discounting each payment.

Suppose you receive $300 at the end of each year for 4 years. The discount rate is 9%.

Using the annuity formula:

$$PV = 300\left[\frac{1-\frac{1}{(1.09)^4}}{0.09}\right] = 971.92$$

Using the long way:

Year	Cash flow	Present value
1	$300	$275.23
2	$300	$252.51
3	$300	$231.66
4	$300	$212.51
Total		$971.92

Same answer. The annuity formula just gets there faster.

8.4.1 Deferred Annuity

Now suppose the $300 payments start at year 3 and continue through year 6. There are still 4 payments, but the annuity does not start immediately.

Step 1: value the 4-payment annuity one period before the first payment, at $t = 2$.

$$PV_2 = 300\left[\frac{1-\frac{1}{(1.09)^4}}{0.09}\right] = 971.92$$

Step 2: discount that $t = 2$ value back to today.

$$PV_0 = \frac{971.92}{(1.09)^2} = 818.04$$

This timing idea matters a lot later when we use terminal values. A shortcut gives a value one period before the first cash flow in the shortcut.

8.4.2 Growing Perpetuity Timing

If the first growing cash flow is $120 at $t = 1$, with $r = 10\%$ and $g = 3\%$:

$$PV_0 = \frac{120}{0.10 - 0.03} = 1{,}714.29$$

If the first growing cash flow is $120 at $t = 4$, the shortcut value is at $t = 3$:

$$PV_3 = \frac{120}{0.10 - 0.03} = 1{,}714.29$$

Then you still need to discount that amount back to today.

8.4.3 Go a little further

Deferred annuities and terminal values are cousins. In both cases, the shortcut value is placed one period before the first cash flow represented by the shortcut. If you remember that sentence, the timing gets much less spooky.

8.5 Lab 5: Bond Pricing With A Premium Bond

Consider a 4-year bond with:

- face value = $1,000,
- annual coupon rate = 10%,
- YTM = 8%,
- semiannual coupons.

Inputs:

$$N = 8, \qquad i = 4\%, \qquad PMT = 50, \qquad FV = 1{,}000$$

Price:

$$PV = 50\left[\frac{1 - \frac{1}{(1.04)^8}}{0.04}\right] + \frac{1{,}000}{(1.04)^8} = 1{,}067.33$$

This bond sells above par because the coupon rate is above the YTM. Investors are willing to pay more than $1,000 because the bond pays coupons that are attractive relative to the required return.

8.5.1 Discount, Par, Premium

Bond	Coupon rate	YTM	Price intuition
Discount bond	6%	8%	below $1,000
Par bond	8%	8%	near $1,000
Premium bond	10%	8%	above $1,000

8.5.2 Calculator Troubleshooting

If the answer looks wrong	Check this first
Price way too high or low	Did you use semiannual N and i?
Coupon seems doubled	Did you use annual coupon instead of semiannual coupon?

If the answer looks wrong	Check this first
Calculator shows negative PV	That is probably sign convention
YTM solve fails	Do your cash flows have opposite signs?

8.5.3 Go a little further

Bond price sensitivity is higher when maturity is longer and coupon rate is lower. That idea is related to duration. You do not need duration for these videos, but it explains why long-term bonds can move a lot when market rates change.

8.6 Lab 6: NPV By Hand Before Excel

Excel is useful, but the logic is not Excel logic. It is timeline logic.

A project has the following cash flows:

Time	Cash flow
0	-$900
1	$250
2	$300
3	$350
4	$400

The discount rate is 11%.

Time	Cash flow	Discount factor	Present value
0	-$900	1.0000	-$900.00
1	$250	0.9009	$225.23
2	$300	0.8116	$243.49
3	$350	0.7312	$255.92
4	$400	0.6587	$263.47

NPV:

$$-900 + 225.23 + 243.49 + 255.92 + 263.47 = 88.12$$

Positive NPV means the project is expected to create $88.12 of value today after earning the 11% required return.

8.6.1 Sensitivity Check

At a 14% discount rate, the same project has:

$$NPV = -900 + \frac{250}{1.14} + \frac{300}{1.14^2} + \frac{350}{1.14^3} + \frac{400}{1.14^4}$$

$$NPV = 23.21$$

The project is still positive, but less attractive. Higher required return means future cash flows are discounted more heavily.

8.7 Lab 7: Terminal Value Timing, One More Time

Suppose a project has explicit cash flows through year 4. After year 4, the project is expected to produce a continuing cash flow of $180 at year 5, growing at 2% forever. The discount rate is 10%.

The first continuing cash flow is at $t = 5$, so the terminal value is at $t = 4$:

$$TV_4 = \frac{180}{0.10 - 0.02} = 2{,}250$$

If the year 4 explicit cash flow is $220, then the total year 4 amount is:

$$220 + 2{,}250 = 2{,}470$$

That full $t = 4$ amount is discounted by four periods:

$$\frac{2{,}470}{(1.10)^4} = 1{,}687.04$$

8.7.1 Common Mistake

Do not place the terminal value at the same date as the first continuing cash flow. The growing perpetuity value sits one period before that first continuing cash flow.

8.7.2 Go a little further

Terminal value is often where valuation becomes judgment-heavy. The formula is simple, but the long-run growth rate and discount rate carry a lot of weight. That is why analysts usually test several growth and discount-rate assumptions instead of trusting one answer too much.

8.8 Mixed Practice Set

Try these before reading the answers.

1. You invest $800 today and receive $1,000 in 3 years. What annual return did you earn?

2. What is the present value of $2,000 received in 5 years at 7%?
3. A 12% APR compounded monthly has what monthly rate?
4. What is the EAR for 12% APR compounded monthly?
5. What is the PV of $150 per year for 4 years at 8%?
6. A perpetuity pays $90 next year. The discount rate is 6%. What is PV?
7. A growing perpetuity pays $90 next year, grows at 2%, and has a 7% discount rate. What is PV?
8. A 5-year, 8% coupon bond with $1,000 face value pays semiannually. What is the semiannual coupon?
9. That same bond has how many semiannual periods?
10. If coupon rate is greater than YTM, is the bond usually above or below par?
11. A project has $CF_0 = -400$, $CF_1 = 180$, $CF_2 = 180$, and $CF_3 = 180$. At 10%, what is NPV?
12. A continuing cash flow begins at $t = 6$. Where is the terminal value placed?

Answers are in the answer key at the back.

9 Exam-Style Walkthroughs

This section is less about new formulas and more about choosing the right move. On exams, the hard part is often not the arithmetic. It is recognizing the cash-flow pattern and the timing.

Relevant videos. S1-01 through S1-11.

9.1 Walkthrough 1: Choose The Tool

Read the cash-flow pattern before calculating.

Problem wording	Pattern	Tool
“One cash flow 6 years from now”	single cash flow	PV or FV
“Same payment every year for 10 years”	fixed repeated cash flow	annuity
“Same payment forever”	level perpetuity	perpetuity
“Payment grows forever”	growing perpetuity	growing perpetuity
“Different cash flow each year”	irregular cash flows	discount each one
“Bond pays coupons and face value”	annuity plus lump sum	bond pricing

This is why the timeline comes first. The timeline tells you the pattern.

9.1.1 Mini Example

You are offered three payments: $100 in year 1, $150 in year 2, and $200 in year 3. The discount rate is 9%.

This is not an annuity because the payments are not the same. Discount each one:

$$PV = \frac{100}{1.09} + \frac{150}{1.09^2} + \frac{200}{1.09^3}$$

$$PV = 372.43$$

9.1.2 Common Mistake

Do not force an annuity formula onto uneven cash flows just because there are multiple payments. “Multiple” does not mean “annuity.” An annuity has equal payments at regular intervals.

9.2 Walkthrough 2: Solving For Time

How long does it take money to double at 7% per year?

Start with:

$$FV = PV(1 + r)^N$$

If money doubles, then $FV/PV = 2$:

$$2 = (1.07)^N$$

Solve for N:

$$N = \frac{\ln(2)}{\ln(1.07)} = 10.24$$

At 7% per year, money doubles in a little over 10 years.

9.2.1 The Rule Of 72

A quick mental shortcut says:

$$\text{doubling time} \approx \frac{72}{\text{interest rate in percent}}$$

At 7%:

$$\frac{72}{7} = 10.29$$

That is close to the exact answer. The Rule of 72 is not the official solution, but it is a nice intuition check.

9.2.2 Go a little further

The Rule of 72 works best for moderate interest rates. It is a mental math shortcut, not a substitute for the logarithm formula when you need precision.

9.3 Walkthrough 3: Bond Prices When YTM Changes

Use the same 5-year, 6% coupon, $1,000 face value bond. Coupons are semiannual, so $PMT = 30$ and $N = 10$.

Annual YTM	Semiannual rate	Approximate price
6%	3%	$1,000.00
8%	4%	$918.89
10%	5%	$845.57

When YTM rises, price falls. When YTM falls, price rises.

That inverse relationship is one of the most important bond intuitions.

9.3.1 Why Price Falls When YTM Rises

The bond's promised cash flows did not change. The coupons are still $30 every six months and $1,000 at maturity. What changed is the return investors require. If investors require a higher return, they will only pay a lower price for the same promised cash flows.

9.4 Walkthrough 4: Terminal Value Sensitivity

Terminal value can move a lot when r or g changes.

Suppose the first continuing cash flow is $250 one year after the terminal date.

Base case:

$$TV = \frac{250}{0.10 - 0.02} = 3{,}125$$

If growth rises to 3%:

$$TV = \frac{250}{0.10 - 0.03} = 3{,}571.43$$

If the discount rate rises to 11%, with growth back at 2%:

$$TV = \frac{250}{0.11 - 0.02} = 2{,}777.78$$

Small-looking assumption changes can have a large effect because the formula represents many future cash flows.

9.4.1 Practical Takeaway

Do not memorize terminal value as a magic number. Treat it as a structured assumption. The growth rate and discount rate deserve a sanity check.

9.5 Walkthrough 5: Translating A Spreadsheet Into A Timeline

For manual NPV, a spreadsheet should make the timeline obvious.

Row	Year 0	Year 1	Year 2	Year 3
Cash flow	-$1,000	$300	$400	$500
Discount factor at 10%	1.0000	0.9091	0.8264	0.7513
Present value	-$1,000.00	$272.73	$330.58	$375.66

NPV:

$$-1{,}000 + 272.73 + 330.58 + 375.66 = -21.03$$

The future cash flows add up to $1,200, but the NPV is negative because timing and required return matter.

9.5.1 Excel Habit

When you copy the discount factor formula across a row, lock the rate cell with `F4`. The time cell should move across the row. The rate cell should stay fixed.

10 Formula Review

This page is not a substitute for understanding the timeline, but it is useful for a final check.

Idea	Formula
Future value	$FV = PV(1+r)^N$
Present value	$PV = FV/(1+r)^N$
Solve for rate	$r = (FV/PV)^{1/N} - 1$
Solve for periods	$N = \ln(FV/PV)/\ln(1+r)$
Ordinary annuity PV	$PV = C[1 - 1/(1+r)^N]/r$
Level perpetuity	$PV = C_1/r$
Growing perpetuity	$PV = C_1/(r-g)$
Manual NPV	$NPV = CF_0 + \sum CF_t/(1+r)^t$

10.0.1 Formula-sheet warning

The formula sheet does not know the timing. You do. Put the cash flows on the timeline first, then choose the formula.

11 Final Review

The whole Series 1 story is simple:

1. Draw the timeline.
2. Choose the signs.
3. Match the rate to the period.
4. Move cash flows to the same date.
5. Add values only after they are on the same date.

Exam-Day Checklist

Before calculating:
- What is the cash-flow perspective? - Is the rate annual, monthly, or semiannual? - Is the first cash flow today or next period? - Are you solving for price, rate, number of periods, or value? - Does your answer make economic sense?

11.1 Short Written Prompts

1. Explain why a bond with a coupon rate below YTM sells below par.
2. Explain why a growing perpetuity formula uses next period’s cash flow.
3. Explain why CF_0 should not be included inside Excel’s $NPV()$ range.

Answers are in the answer key at the back.

12 Answer Key

Use this section after you have made an honest attempt. The point is not just to check whether the number is right. The point is to notice the move: timeline, rate, sign, formula, interpretation.

12.1 Timelines And Sign Convention

1. Today is -250; year 2 is $+300$.
2. Today is $+1{,}000$; year 1 is $-1{,}100$.

12.2 Present Value, Future Value, And Rates

1. $FV = 500(1.06)^4 = \$631.24$.
2. $PV = 750/(1.10)^2 = \$619.83$.
3. $6\%/12 = 0.5\%$ per month.

12.3 Solving For Unknowns And Repeated Cash Flows

1. $PV = 200[1 - 1/(1.10)^3]/0.10 = \$497.37$.
2. $PV = 50/0.05 = \$1{,}000$.
3. C_1 happens next period, not today.

12.4 Bonds

1. The semiannual coupon is $\$1{,}000 \times 8\%/2 = \$40$.
2. A 3-year semiannual bond has $3 \times 2 = 6$ periods.
3. Below par. The coupon rate is lower than the market-required return.

12.5 Irregular NPV And Terminal Value

1. $NPV = -100 + 60/1.10 + 60/(1.10)^2 = \$4.13$.
2. The terminal value is placed at $t = 4$, one period before the first continuing cash flow at $t = 5$.
3. The valuation is usually overstated because the terminal value is not discounted enough.

12.6 Mixed Practice Set

1. $r = (1{,}000/800)^{1/3} - 1 = 7.72\%$.
2. $PV = 2{,}000/(1.07)^5 = \$1{,}425.97$.
3. $12\%/12 = 1\%$ per month.
4. $(1.01)^{12} - 1 = 12.68\%$.
5. $PV = 150[1 - 1/(1.08)^4]/0.08 = \$496.81$.
6. $PV = 90/0.06 = \$1{,}500$.
7. $PV = 90/(0.07 - 0.02) = \$1{,}800$.
8. The semiannual coupon is $\$1{,}000 \times 8\%/2 = \$40$.
9. A 5-year semiannual bond has $5 \times 2 = 10$ periods.
10. Above par.
11. $NPV = -400 + 180/1.10 + 180/(1.10)^2 + 180/(1.10)^3 = \$47.63$.

12. At $t = 5$.

12.7 Short Written Prompts

1. The bond's coupons are low relative to the required return, so investors pay less than face value to earn the market yield.
2. The perpetuity value is measured one period before the first continuing cash flow.
3. Excel discounts the first value in the range by one period, but CF_0 is already today.

13 Glossary And Index Of Key Ideas

This glossary is here for review. If a term feels fuzzy, go back to the matching section and work one example before moving on.

Annuity. A series of equal cash flows paid at regular intervals for a fixed number of periods. In this Boot Camp, we usually use ordinary annuities, where the first payment arrives one period from now.

APR. Annual percentage rate. An APR is a quoted annual rate. If compounding occurs more than once per year, convert the quote into the correct period rate before using it on a timeline.

Bond price. The present value of the bond's promised coupons plus the present value of the face value paid at maturity.

Cash-flow sign convention. The rule that cash inflows and outflows should have opposite signs from one chosen perspective. If you are the investor, money you pay today is usually negative and money you receive later is usually positive.

Compounding. The process of earning interest on interest. Compounding is why a monthly compounded APR has an effective annual rate that can be slightly higher than the quoted APR.

Coupon. The periodic interest payment on a bond. For a semiannual bond, the coupon paid every six months is the annual coupon rate times face value, divided by two.

Discount factor. The number used to convert a future cash flow into present value. For a cash flow at time t, the discount factor is:

$$\frac{1}{(1+r)^t}$$

The present value is the future cash flow times the discount factor.

Discount rate. The required return used to move future cash flows back to today. Higher discount rates make future cash flows worth less today.

EAR. Effective annual rate. The actual annual rate after accounting for compounding within the year.

Face value. The amount paid back at bond maturity, often $1,000 in class examples.

Future value. The value of money at a future date after compounding forward:

$$FV = PV(1+r)^N$$

Growing perpetuity. A cash flow stream that begins one period from now and grows at a constant rate forever. Its value one period before the first cash flow is:

$$PV = \frac{C_1}{r-g}$$

Level perpetuity. A constant cash flow that begins one period from now and continues forever. Its value one period before the first cash flow is:

$$PV = \frac{C_1}{r}$$

Net present value. The value today of all project cash flows, including the time-zero investment:

$$NPV = CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}$$

Ordinary annuity. An annuity with payments at the end of each period. Most introductory finance annuity problems are ordinary annuities unless stated otherwise.

Par. A bond priced at face value, usually around $1,000 in these examples.

Premium bond. A bond priced above face value, usually because its coupon rate is above the market-required return.

Present value. The value today of a future cash flow:

$$PV = \frac{FV}{(1+r)^N}$$

Terminal value. The value at the end of an explicit forecast period of cash flows expected after that date. With a growing perpetuity, the terminal value is placed one period before the first continuing cash flow.

Yield to maturity. The discount rate that makes the present value of a bond's promised cash flows equal its observed price.